A method for the analysis of respiratory sinus arrhythmia using continuous wavelet transforms

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***Abstract*— A continuous wavelet transform-based method is presented to study the non-stationary strength and phase delay of the respiratory sinus arrhythmia (*RSA*). RSA is the cyclic variation of instantaneous heart rate at the breathing frequency. In studies of cardio-respiratory interaction during sleep, paced breathing or postural changes, low respiratory frequencies and fast changes can occur. Comparison on synthetic data presented here shows that the proposed method outperforms traditional short-time Fourier-transform analysis in these conditions. On the one hand, wavelet analysis presents a sufficient frequency-resolution to handle low respiratory frequencies, for which time-frames should be long in Fourier-based analysis. On the other hand, it is able to track fast variations of the signals in both amplitude and phase, for which time-frames should be short in Fourier-based analysis.**

***Index Terms*—Continuous wavelet transform, cardio-respiratory interaction, heart rate variability, respiratory sinus arrhythmia.**

I. Introduction

THE heart rate variability (*HRV*) is traditionally divided in very low frequency (*VLF*), low frequency (*LF*), and high frequency (*HF*) components, the frequency bands of which are respectively $[0.003Hz, 0.04Hz]$, $[0.04Hz, 0.15Hz]$, and $[0.15Hz, 0.5Hz]$ ([1]). In the HF band, an oscillation can usually be observed at the breathing frequency. It is called the respiratory sinus arrhythmia (*RSA*). The study of RSA has produced extensive literature both concerning its causes [2], [3], and the methodology to determine it [1]. It is generally accepted that RSA amplitude is a non-invasive marker of the activity of the parasympathetic nervous system [3], [4] and can therefore be used to infer relative changes in parasympathetic cardiac tone. RSA phase delay is a measure of the time delay between respiratory cycles and RSA. It is an indirect, non-invasive measure of the integration time of the cardio-respiratory interaction, and can be estimated by frequency [5], [6] as well as time domain methods [7].

Dynamic analysis of HRV is traditionally performed by means of short-time Fourier transform (*STFT*), following established guidelines [1]. However, STFT analysis is limited by its time-frequency resolution trade-off: a long window gives a better frequency-resolution, and a short window a better time-resolution. The window-length optimization is difficult for short or non-stationary time series. In these cases, outputs are averaged over different states or conditions, and transients blurred. Therefore, non-stationary spectral methods have been implemented, based on time-variant autoregressive models [8], Wigner-Ville Distribution [8], [9], selective discrete Fourier transform [10], or discrete or continuous wavelet transform, using various mother wavelets (e.g. Daubechies 4 [11], harmonic wavelet [12]).

The aim of this article is to present an analysis method of the cardio-respiratory interaction based on continuous wavelet transforms (*CWT*). It is compared to STFT analysis on synthetic data, with a view to handle low breathing frequencies and fast variations. This is of particular relevance in the study of RSA dynamics during sleep, where respiratory frequencies below $0.2Hz$ (e.g [13]) occur simultaneously to fast variations in RSA, due to changes in sleep stage and corresponding sympatho-vagal balance [14]. Sudden RSA changes, accompanied by low breathing frequencies, are also present in paced breathing protocols [7], and postural changes (tilt) experiments [8], [10].

II. Methods

A. Algorithm

The algorithm calculates the gain and phase delay of the RSA, based on CWTs. The analysis is limited to time-intervals where the estimated respiratory frequency and the estimated frequency of the main peak in the HF band of HRV are equal within $\pm 0.02Hz$.

a) Continuous wavelet transform analysis: The time-dependent power spectra of the respiratory and HRV signal are calculated by means of CWTs.

The continuous wavelet transform of a signal $x(t)$ is defined as

$$CWT(t,\lambda) = \int_{-\infty}^{+\infty} x(u)\, \frac{1}{\sqrt{\lambda}} \psi^* \left(\frac{u-t}{\lambda} \right) du, \qquad (1)$$

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 Manuscript received May 11, 2007; final revision October 29, 2007.

where $\psi(t)$ is the mother wavelet, and $CWT(\lambda, t)$ the wavelet transform coefficient for scale λ at time t. The time- and frequency-resolution of the wavelet $\psi_\lambda(t)$ are defined as $4\sigma_t$ and $4\sigma_f$, respectively. σ_t and σ_f are the standard deviation of $|\psi_\lambda(t)|^2$ and $\left|\hat{\psi}_\lambda(f)\right|^2$, where $\psi_\lambda(t) = \frac{1}{\sqrt{\lambda}}\psi\left(\frac{u-t}{\lambda}\right)$ and $\hat{}$ symbolizes the Fourier transform.

The amplitude and phase of the complex CWT coefficients obtained using an analytical mother wavelet are estimates of the envelope and instantaneous phase of the spectral components of the signal in the frequency-band centred on the central frequency of the wavelet [15]. Here, the complex Morlet wavelet [15] is used because it is a Gaussian-shaped analytical wavelet. This shape optimizes the product of the time- and frequency-resolutions of the wavelet.

The time-resolution in the HF band should be close to $30s$, the duration of standard sleep stage scoring windows. On the other hand, the frequency-resolution in the VLF and LF bands should be optimized. To satisfy both criteria, parameter f_0, the central frequency of the Morlet mother wavelet, is set to $\frac{15}{2\pi}$ Hz in this study. The time-resolution then varies from $45s$ to $14s$ in the HF band, and the frequency-resolution varies from $7mHz$ to $28mHz$ in the LF band.

From the CWT of the respiratory volume signal, the respiratory frequency $f_{resp}(t)$ is estimated by the frequency corresponding to the maximal CWT amplitude for each time-step. The corresponding phase $\Phi_{resp}(t)$ is given by the CWT phase at frequency $f_{resp}(t)$. The energy $E_{resp}(t)$ is estimated by the surface of the peak, using a Gaussian approximation of the peak's shape. The frequency $f_{HF}(t)$, corresponding phase $\Phi_{HF}(t)$ and energy $E_{HF}(t)$ of the HF peak of the HRV signal is estimated in a similar way from the CWT of the HRV signal in the HF band.

The amplitude ratio and phase delay (in seconds) between the respiratory signal and RSA component of the HRV signal are given by $AR(t) = \frac{E_{HF}(t)}{E_{resp}(t)}$ and $\Delta\Phi_t(t) = \frac{\Phi_{resp}(t) - \Phi_{URRI}(t)}{2\pi f_{resp}}$.

B. Synthetic data

Synthetic cardiac and respiratory data with varying phase, amplitude and frequency are created to test the algorithm, and to compare it to STFT-based algorithms.

The respiratory volume signal $S_{resp}(t)$ is synthesized by means of a sinusoid of frequency f_{resp}, amplitude a_1 and phase Φ_{t0}, to which white noise was added (Eq. 2). All parameters can vary in time, corresponding to changes in breathing frequencies, tidal volume and phase drifts or "resets" of the respiratory cycle.

Instantaneous heart period variability, represented by the uniformly resampled RR-interval time series $S_{URRI}(t)$, is synthesized by summing a constant b_0 (representing the average heart beat interval) with three sinusoids of amplitudes b_{VLF}, b_{LF}, b_{HF}, and of frequencies f_{VLF}, f_{LF}, f_{HF} (Eq. 3). Because the HF component represents the RSA, f_{HF} is set equal to f_{resp}. A phase parameter $\Phi_{t0} + \Delta\Phi_t$ is added in the HF component to model the phase difference between heart rate and respiratory signals. White noise is added to the phase, and to the signal.

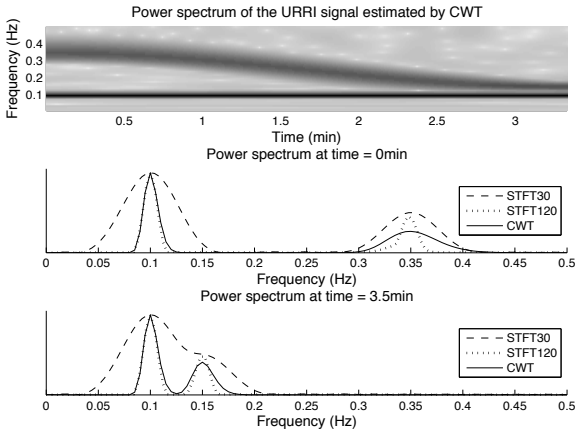

Fig. 1. Synthetic data with a respiratory frequency varying from $0.35Hz$ to $0.15Hz$: the first panel shows the power spectrum of the URRI signal estimated by CWT. The ridge at the respiratory frequency can be distinguished from the low-frequency component at $0.1Hz$ for all time-steps. The second and third panels show the normalized power spectra of the synthetic HRV signal, estimated by STFT30 (dashed line), STFT120 (dotted line) and CWT (plain line), at time $0min$ (second panel) and $3.5min$ (third panel). In the second panel, for time $0min$, all algorithms are able to distinguish between the two peaks. In the third panel, for time $3.5min$, only STFT120 and CWT give acceptable results.

The synthetic respiratory and heart-rate interval signals are thus given by

$$S_{resp}(t) = a_1 cos(2\pi f_{resp}(t + \Phi_{t0})) + noise(t), \quad (2)$$

$$S_{URRI}(t) = b_0 + b_{VLF} cos(2\pi f_{VLF} t) + b_{LF} cos(2\pi f_{LF} t) + b_{HF} cos(2\pi f_{resp}(t + \Phi_{t0} + \Delta\Phi_t + noise_\Phi(t))) + noise(t). \quad (3)$$

III. Results

The performance of the CWT-based algorithm is compared to the results obtained via short-time Fourier transform-based methods, using 30 s and 120 s-long Hamming windows, *STFT30* and *STFT120*. To study the dynamics of the cardio-respiratory interaction, the major limitations arise from the ability of the method to detect the RSA peak, and to track fast changes. Noise is not a major issue, because the interpolated instantaneous heart period (RR-interval) time series used in HRV analysis usually presents low noise levels [1]. Therefore, the algorithms are compared with regard to the value of the respiratory frequency and to time-varying phase delays.

b) Time-varying respiratory frequency: The sensitivity of the methods to respiratory frequency changes is evaluated on synthetic data with a sinusoidally modulated respiratory frequency. The variation range covers the frequency-interval of normal breathing $[0.15Hz, 0.35Hz]$. $f_{resp}(t) = \overline{f_{resp}}(1 + A_{mod} cos(2\pi f_{mod} t))$, where $\overline{f_{resp}} = 0.25Hz$, $A_{mod} = 0.4$ and $f_{mod} = 0.0025Hz$. The two low-frequency components of the URRI signal are modelled by a unique peak at $0.1Hz$. The other parameters of Eq. 2 and 3 are $a_1 = 0.2$, $b_0 = 1s$, $b_{VLF} = 0$, $b_{LF} = 0.06s$, $f_{LF} = 0.1Hz$, $b_{HF} = 0.03s$, $\Phi_{t0} = 0$, $\Delta\Phi_t = 0$, $noise_\Phi = 0$, and $noise(t)$ is white noise with a maximal amplitude of $0.02s$.

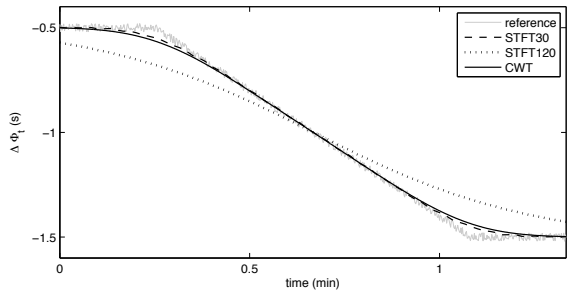

Fig. 2. Synthetic data with a phase delay varying from $-0.5s$ to $-1.5s$, with white noise of maximal amplitude $0.02s$ added to the phase: phase delay $\Delta\Phi_t$ estimated by STFT30 (dashed line), STFT120 (dotted line) and CWT (plain line). STFT30 and CWT are able to track similarly the $\Delta\Phi_t$ variations, while STFT120 performs less accurately.

The upper panel of Fig. 1 shows the power spectrum of the URRI series estimated by the CWT method. The ridge in the power spectrum at the respiratory frequency can be easily distinguished from the low-frequency component (at $0.1Hz$) at all time-steps. The lower panels show the power spectra obtained by the STFT30, STFT120 and CWT methods, at the beginning and end of the signal. When the frequencies of the two peaks are far from each other, the peaks can be separated by all methods. However, when the respiratory frequency gets closer to the $LF-HF$ frontier, STFT30 fails, because the frequency resolution cannot separate the HF-peak from the neighboring LF-peak.

c) Time-varying phase delay: The ability to track phase delays is evaluated on synthetic data with a constant respiratory frequency $f_{resp}=0.3Hz$ and a time-varying phase delay. The phase delay expressed in seconds $\Delta\Phi_t(t)$ varies linearly from $-0.500s$ to $-1.500s$ in $50s$, between two constant plateaus. $noise_\Phi$ is white noise with a maximal amplitude of $0.02s$. The remaining parameters of Eq. 2 and 3 are $a_1=0.2$, $\Phi_{t0}=0$, $b_0=1s$, $b_{VLF}=0.07s$, $f_{VLF}=0.04Hz$, $b_{LF}=0.06s$, $f_{LF}=0.1Hz$, $b_{HF}=0.06s$, and $noise(t)=0$.

Fig. 2 shows the phase delay $\Delta\Phi_t(t)$ estimated by STFT30, STFT120 and CWT, together with the reference. STFT30 and CWT are able to track similarly the $\Delta\Phi_t$ variations, while STFT120 performs less accurately. Indeed, the maximal estimation errors are $0.059s$, $0.064s$ and $0.186s$ for STFT30, CWT and STFT120, respectively. The poor results of STFT120 are due to its low time-resolution: results are averaged over the window-length.

The algorithm's ability to track fast changes depends on its time-resolution. It can be quantified via the rise time of the obtained phase response to an imposed step in the phase delay. For STFT30 and STFT120, the rise time from 10% to 90% of the final value is independent of the respiratory frequency, and equal to $15s$ and $55s$, respectively. For CWT, it is equal to $15s$ at $f_{resp}=0.35Hz$, and $34s$ at $f_{resp}=0.15Hz$. These data confirm that CWT and STFT30 perform similarly at high respiratory frequencies, while STFT120 performs worse. Although CWT does not perform as well for low as for high respiratory frequencies, it still outperforms STFT120. For low respiratory frequencies, STFT30 theoretically performs best in tracking changes. However, it is unable to isolate the HF peak.

IV. Discussion and conclusion

A continuous wavelet transform-based algorithm is presented for the analysis of respiratory sinus arrhythmia. The proposed method is compared on synthetic data to short-time Fourier transform-based analyses using short or long time-windows ($30s$ and $120s$). These window-lengths have been chosen because the first gives the desired time-resolution in the HF band, while the second is a standard compromise for studying HRV.

Results on synthetic data show that neither STFT30 nor STFT120 are able to simultaneously and accurately identify low respiratory frequencies and transients. On the one hand, Fourier-based analysis with a short window cannot distinguish between LF and HF peaks in the URRI signal, when the respiratory frequency drops below $0.2Hz$. On the other hand, Fourier-based analysis with a long window is not able to track fast phase delay variations.

Results obtained with the CWT-based method on synthetic data show that the method is able to handle both limitations of the STFT-based algorithms. This is explained by the intrinsic variation of the trade-off between time- and frequency-resolutions in CWTs: a low time- and high frequency-resolution are used to analyze low frequencies, which allows the detection of the HF component of HRV even for low respiratory frequencies, and a high time- and low frequency-resolution to analyze high frequencies, which allows the tracking of fast variations. The proposed method is therefore better suited than Fourier-based analyses for the study of cardio-respiratory interaction dynamics, when low breathing frequencies are present.

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